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The low energy problem on astrophysics braneworld*

*Jorge Ovalle***

Departamento de Física, Universidad Central de Venezuela, Caracas, Venezuela.

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Abstract

Considering an anisotropic and non uniform stellar distribution in the context of braneworld, an exact analytic solution to Einstein's field equations is found, however it is not possible to regain the low energy limit. To find the source of this "low energy limit problem" a careful analysis on an isotropic and no uniform distribution is performed. It is shown that the source of the problem is the KK corrections to the field equations of general relativity.

Key words: braneworld, general relativity.

El problema a bajas energías en el mundo brana astrofísico

Resumen

Considerando una distribución anisótropa y no uniforme en el contexto del mundo brana, una solución analítica y exacta de las ecuaciones de campo de Einstein es encontrada, aunque no es posible recuperar el límite a bajas energías. Para determinar la fuente del problema del límite a bajas energías, se realiza un análisis minucioso sobre una distribución isótropa y no uniforme. Se encuentra que la fuente del problema son las correcciones KK de las ecuaciones de campo de la relatividad general.

Palabra claves: mundo brana, relatividad general.

Introduction

The Superstring/M-theory is considered one of the most promising candidate theories of quantum gravity. It describe gravity as a high dimensional interaction which becomes effectively four dimensional at low enough energies. This theory has inspired the construction of braneworld models, in which the standard model gauge fields are confined to our observable universe (the brane), while gravity propagates in all spatial dimensions (the bulk) (1).

The implications of the braneworld theory on general relativity have been extensively investigated (2-4), most of theme on cosmological scenarios (5-11). The studies on astrophysics braneworld (12-15) are limited, however it is well known that gravitational collapse could produce very high energies, where the braneworld corrections to general relativity would become significant.

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** Autor para la correspondencia. E-mail: jovalle@usb.ve.

Any solution found in the context of astrophysics braneworld must have the well known general relativity limit, however this problem has not been fully resolved. Due to the highly non lineal contributions of pure high energy and KK terms, to find a solution to field equations is very difficult. For this reason demanding the general relativity limit, even in the simplest case, is an additional problem which solution is very complex.

In order to clarify this fundamental aspect on stellar braneworld, in this paper two possible static scenarios are considered. In the case of an anisotropic spherical distribution with non uniform density, a simple analytic solution is found. In this case the low energy limit is not regained. The source of this "low energy limit problem" is identified as the KK corrections on the brane. The final scenario considered is the isotropic and no homogeneous stellar distribution, where the source of the low energy limit problem is clearly identified.

The field equations and matching conditions

The Einstein field equations on the brane may be written as a modification of the standard field equations (16)

$$G_{\mu\nu} = -8\pi T^{T}_{\mu\nu} - \Lambda g_{\mu\nu}, \qquad [1]$$

where the energy-momentum tensor has new terms carrying bulk effects onto the brane:

$$T_{\mu\nu} \rightarrow T^{\mathrm{T}}_{\mu\nu} = T_{\mu\nu} + \frac{6}{\lambda_b} S_{\mu\nu} + \frac{1}{8\pi}\xi_{\mu\nu} \qquad [2]$$

Here λ_b is the brane tension and Λ the cosmological constant on the brane. We consider a spherically symmetric static distribution. The line element is given in Schwarzschild-like coordinates by

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad [3]$$

where ν and λ are functions of r.

The metric [3] has to satisfy [1]. In our case with Λ=0 we have:

$$-8\pi\left(\rho + \frac{1}{\lambda_b}\left(\frac{\rho^2}{2} + 6\mathcal{U}\right)\right) = -\frac{1}{r^2} + e^{\lambda}\left(\frac{1}{r^2} - \frac{\lambda_1}{r}\right), \qquad [4]$$

$$-8\pi\left(-P - \frac{1}{\lambda_b}\left(\frac{\rho^2}{2} + \rho P + 2\mathcal{U}\right) + \frac{\mathcal{P}}{\lambda_b}\right) = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda_1}{r}\right), \qquad [5]$$

$$-8\pi\left(-P - \frac{1}{\lambda_b}\left(\frac{\rho^2}{2} + \rho P + 2\mathcal{U}\right) - \frac{\mathcal{P}}{2\lambda_b}\right) = \frac{1}{4}e^{-\lambda}\left[2\upsilon_{11} + \upsilon_1^2 - \lambda_1\upsilon_1 + 2\frac{(\upsilon_1 - \lambda_1)}{r}\right] \qquad [6]$$

$$P_1 = -\frac{\upsilon_1}{2}(\rho + P) \qquad [7]$$

where *f1=df/dr*. The general relativity (low energy limit) is regained when $\lambda^{-1} \rightarrow 0$ and [7] becomes a linear combination of [4]-[6].

Using the conservation equation

$$\nabla^{\mu}T^{T}_{\mu\upsilon} = 0 \qquad [8]$$

we obtain

$$\mathcal{U}_1 + \frac{\nu_1}{2}4\mathcal{U} - 2\mathcal{P}_1 - \frac{\nu_1}{2}2\mathcal{P} - 6\frac{\mathcal{P}}{r} = -(\rho + P)\rho_1\frac{k^4}{2} \qquad [9]$$

This equation is a linear combination of the field equations [4]-[6] and [7], so there is not new information from it. However we can learn that the density gradients are a source for Weyl stresses in the interior.

$$\rho = \rho(r) \rightarrow \mathcal{U} \quad and/or \quad \mathcal{P} \neq 0 \qquad [10]$$

The Israel-Darmois matching conditions at the stellar surface Σ give

$$\left[G_{\mu\nu}r^{\nu}\right]_{\Sigma} = 0 \qquad [11]$$

where $[f]_{\Sigma} \equiv f(r)\big|_{\alpha^+} - f(r)\big|_{\alpha^-}$ Using [11] and the field equation [1] with Λ=0 we have

$$\left[T^{T}_{\mu\nu} r^{\nu}\right]_{\Sigma} = 0, \qquad [12]$$

which leads to

$$\left[\left(P + \frac{1}{\lambda_b}\left(\frac{\rho^2}{2}\rho P + 2\mathcal{U}\right) - \frac{\mathcal{P}}{\lambda_b}\right)\right]_{\Sigma} = 0, \qquad [13]$$

this takes the final form

$$P_\alpha + \frac{1}{\lambda_b}\left(\frac{\rho_\alpha^2}{2} + \rho_\alpha P_\alpha + 2\mathcal{U}_\alpha^-\right) - \frac{\mathcal{P}_\alpha^-}{\lambda_b} = \frac{2\mathcal{U}_\alpha^-}{\lambda_b} - \frac{\mathcal{P}_\alpha^+}{\lambda_b}, \qquad [14]$$

where $f_a \equiv f(r)\,|_{r=a}$. The equation [14] gives the general matching condition for any static spherical star on brane world. When $\lambda^{-1} \to 0$ we obtain the well known matching condition $P_a=0$. In the particular case of the Schwarzschild exterior solution $U^+=P^+=0$, the matching condition [14] becomes:

$$P_\alpha + \frac{1}{\lambda_b}\left(\frac{\rho_\alpha^2}{2} + \rho_\alpha P_\alpha + 2\mathcal{U}_\alpha^-\right) - \frac{\mathcal{P}_\alpha^-}{\lambda_b} = 0 \qquad [15]$$

We may see that the matching conditions do not have a unique solution on the brana (12).

Anisotropic and no uniform stellar distributions: a solution

In the case of an anisotropic and non homogeneous static distribution, we have an overdetermined system of equations [4]-[7] which solution must be determined giving additional information.

From the field equations [5] and [6] we obtain

$$8\pi\frac{\mathcal{P}}{\lambda_b} = \frac{2}{3}\left(G_2^2 - G_1^1\right) \qquad [16]$$

and

$$\frac{6\mathcal{U}}{\lambda_b} = -\frac{3}{\lambda_b}\left(\frac{\rho^2}{2} + \rho P\right) + \frac{1}{8\pi}\left(2G_2^2 + G_1^1\right) - 3P, \qquad [17]$$

with

$$G_1^1 = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2}\frac{\upsilon_1}{r}\right), \qquad [18]$$

$$G_2^2 = -\frac{1}{r^2} + e^{-\lambda}\left[2\upsilon_{11} + \upsilon_1^2 - \lambda_1\upsilon_1 + 2\frac{(\upsilon_1 - \lambda_1)}{r}\right]. \qquad [19]$$

Now using [17] in the field equation [4] we have

$$-\lambda_1 e^{-\lambda} + e^{-\lambda}\left(\frac{\upsilon_{11} + \upsilon_1^2/2 + 2\upsilon_1/r + 2/r^2}{\upsilon_1/2 + 2/r}\right)$$
$$= \frac{2}{r^2(\upsilon_1/2 + 2/r)} - 8\pi\frac{\left(\rho - 3P - \frac{1}{\lambda_b}\rho(\rho + 3P)\right)}{(\upsilon_1/2 + 2/r)}, \qquad [20]$$

hence finding υ, ρ and P satisfying the conservation equation

$$P_1 = -\frac{\upsilon_1}{2}(\rho + P) \qquad [21]$$

we would be able to find λ, U and P by [16], [17] and [20].

A simple solution to consider is

$$\rho = A + Br^{k/2};\ \ P = -A - \frac{B}{2}r^{k/2};\ \ e^{\upsilon/2} = Cr^{k/2} \qquad [22]$$

Where $k<0$, A, B and C are constants to be determined by matching conditions.

The solution [22] leads

$$m(r) = M(r) + \frac{4\pi r^3}{\lambda_b}f(r) + g(r), \qquad [23]$$

where

$$M(r) = M\left(\frac{r}{\alpha}\right)^{3+k/2} + \frac{4\pi r^3}{3}A\left[1 - \left(\frac{r}{\alpha}\right)^{k/2}\right], \qquad [24]$$

here M is the total mass of the distribution and a its radius,

$$f(r) = \frac{2}{3}A^2 + \frac{5}{2}\frac{AB}{(3+k/2)}T^{k/2} + \frac{1}{2}\frac{B^2}{(3+k)}r^k \qquad [25]$$

and *g(r)* a function which *must* vanish when $\lambda^{-1}\to 0$, however this condition does not hold and the GR limit is not regained. It is necessary to carry out a more careful analysis. This will be addressed in the next section.

Isotropic and non uniform stellar distributions

In order to find the source of the "low energy limit problem" found in the previous section, we will consider an isotropic and no homogeneous stellar distribution. In this case

$$G_1^1 = G_2^2 \qquad [26]$$

and [17] becomes

$$\frac{6\mathcal{U}}{\lambda_b} = \frac{3}{\lambda_b}\left(\frac{\rho^2}{2} + \rho P\right) + \frac{3G_1^1}{8\pi} - 3P. \qquad [27]$$

Now using [27] in the field equation [4] we have the following differential equation for λ(r):

$$-\lambda_1 e^{-\lambda} + e^{-\lambda_1}\left(\frac{4}{r} + 3\upsilon_1\right) = \frac{4}{r} + 8\pi r(3P - \rho)$$
$$+\frac{8\pi r}{\lambda_b}(\rho^2 + 3\rho P), \qquad [28]$$

which formal solution is

$$e^{-\lambda} = r^{-4}e^{-3\upsilon}\left(\int r^4 e^{3\upsilon}\left[\frac{4}{r} + 8\pi r(3P-\rho)\right.\right.$$
$$\left.\left.+\frac{8\pi r}{\lambda_b}(\rho^2 + 3\rho P)\right] + c_0\right) \qquad [29]$$

however this solution has "mixed" the high energy contribution, in consequence finding how the low energy limit problem originates becomes very complicated. To clarify this point let us rewrite the differential equation [28] by

$$\left[-\lambda_1 e^{-\lambda} + \frac{e^{-\lambda_1}}{r} - \frac{1}{r} + 8\pi r\rho\right] + \left[3e^{-\lambda}\left(\frac{1}{r} + \upsilon_1\right) - \frac{3}{r}\right.$$
$$\left.-8\pi r 3\rho - \frac{8\pi r}{\lambda_b}(\rho^2 + 3\rho P)\right] = 0, \qquad [30]$$

where the right bracket has the *KK* and high energy contribution to the differential equation of λ(r). The solution now can be written in a way that allows to see clearly the effects of high energy contribution

$$e^{-\lambda} = \mu + \frac{e^{-3\upsilon}}{r^4}\int_0^r r^4 e^{3\upsilon}\left[3rH(r) + \frac{8\pi r}{\lambda_b}(\rho^2 + 3\rho P)\right], \qquad [31]$$

where

$$\mu \equiv 1 - \frac{8\pi}{\rho}\int_0^r r^2 \rho dr \qquad [32]$$

and

$$H(r) \equiv \left[8\pi P + \frac{1}{r^2} - \mu\left(\frac{1}{r^2} + \frac{\upsilon_1}{r}\right)\right]. \qquad [33]$$

We know that the function *H(r)* vanishes when $\lambda^{-1}\to 0$ [5], so the solution found [31] have the well known low energy limit:

$$e^{-\lambda} = 1 - \frac{8\pi}{r}\int_0^r r^2 \rho dr, \qquad [34]$$

however to find a regular solution for the system [4]-[7] with P=0 subject to [31] is not easy at all.

Conclusions

In the context of the astrophysics braneworld, a simple analytic solution to an anisotropic and no homogeneous stellar distribution has been found. However the low energy limit for this specific solution is not regained. The source of this low energy limit problem was identified clearly as the KK contributions carrying bulk effects on the brane. Hence, it was necessary to perform a more careful analysis of this problem in order to find a consistent solution with respect to well known low energy limit.

In the case of an isotropic and no homogeneous stellar distribution, the general solution of the geometric function $\l(r)$ was written on a way which allows to see clearly the effects of high energy contribution. The source of the low energy limit problem was clearly identified.

Finally, we want to emphasize that any study on static stellar behavior, in the context of braneworld, must take in account the way identified here to find a consistent solution with respect to well known low energy limit. This is the more relevant contribution of this work.

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