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The logical relations of opposition and how to teach them¹

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Abstract

Based on the mental models theory, in this paper, I try to identify which of the relations of opposition in Aristotelian logic can be often difficult to understand for the students. In this way, assuming the distinction that the theory proposes between canonical and noncanonical models, I analyze all of the relations of opposition in order to detect which of them require to manipulate canonical models and, therefore, more cognitive effort. In addition, I comment on the kinds of sentences that should be used as examples to make the learning process of the problematic relations easier.

Keywords: Aristotelian logic; mental models; quantified sentences; relations of opposition; semantic possibilities.

Las relaciones lógicas de oposición y cómo enseñarlas

Resumen

Basándome en la teoría de los modelos mentales, en este trabajo, trato de identificar cuáles de las relaciones de oposición de la lógica aristotélica pueden ser generalmente difíciles de entender para los estudiantes. De este modo, asumiendo la distinción que la teoría propone entre modelos canónicos y no canónicos, analizo todas las relaciones de oposición con el fin de detectar cuáles de ellas requieren manipular modelos canónicos y, por tanto, un mayor esfuerzo cognitivo. Además, comento los tipos de sentencias que deberían ser utilizados como ejemplos para lograr que el proceso de aprendizaje de las relaciones problemáticas sea más sencillo.

Palabras Clave: lógica aristotélica; modelos mentales; sentencias cuantificadas; relaciones de oposición; posibilidades semánticas.

1. INTRODUCTION

As it is well known, Aristotelian logic argues that there are relations of oppositions between the different kinds of quantified sentences. The educational problem with this is that such relations, which were expressed later in a square that was named ‘Square of Opposition,’ are not always easy to understand and learn for the students, especially if they are high school students.

However, a current psychological theory addressing reasoning, the mental models theory (e.g., Johnson-Laird, 2004, 2006, 2010, 2012; Khemlani, Orenes, & Johnson-Laird, 2012, 2014; Khemlani, Lotstein, Trafton, & Johnson-Laird, 2015; Oakhill & Garnham, 1996; Orenes & Johnson-Laird, 2012), can explain why this circumstance happens and provide ideas and resources to teach the relations in a simple way, i.e., in a way that makes such relations easy to understand. Thus, it can be thought that the mental models theory can indicate to professors and teachers how to achieve that their students better understand the Square of Opposition.

To show all of this, I will begin by accounting for in details the quantified sentences involved in the square and the relations that Aristotelian logic attributes to them. Then, I will describe the main theses of the mental models theory related to those issues. Finally, I will explain how, by means of the methodological resources of the mental models theory, the relations harder to understand can be identified and the best way to teach these latter relations can be found.

2. THE RELATIONS OF OPPOSITION IN ARISTOTELIAN LOGIC

The kinds of sentences involved in the Square of Opposition are well known. They are four:

- Universal affirmative sentences.
- Universal negative sentences.
- Particular affirmative sentences.
- Particular negative sentences.

Several examples of these kinds of sentences are to be found in different Aristotelian passages. Thus, in *De Interpretatione* (Περὶ Ἑρμηνείας) 7, 17b18-20, Aristotle mentions these ones:

-Πᾶς ἄνθρωπος λευκός, that is, in English, “every man is white.” This is Boger’s (2004: 133) translation, but, of course, other translations are possible, e.g., ‘all of the men are white.’ In any case, this example corresponds to the universal affirmative sentences. So, it can be said that the structure of this type of sentence is ‘every S is P’ (or ‘all of the S are P’), where ‘S’ refers to the subject of the sentence and ‘P’ to its predicate.

-Ἔστι τις ἄνθρωπος λευκός, that is, in English, “Some man is white.” This is Boger’s (2004: 133-134) translation, but, of course, other translations of this sentence are possible too, e.g., ‘some of the men are white.’ In any case, this example corresponds to the particular affirmative sentences. So, it can be said that the structure of this type of sentence is ‘some S is P’ (or ‘some of the S are P’).

-Οὐδεὶς ἄνθρωπος λευκός, that is, in English, “no man is white.” This is Boger’s (2004: 133) translation, but, of course, other translations of this sentence are possible as well, e.g., ‘none of the men is white.’ In

any case, this example corresponds to the universal negative sentences. So, it can be said that the structure of this type of sentence is ‘no S is P’ (or ‘none of the S is P’).

-Οὐ πᾶς ἄνθρωπος λευκός, that is, in English, “not every man is white.” This is Boger’s (2004: 133) translation, but, of course, other translations of this sentence are also possible, e.g., ‘some of the men are not white.’ In any case, this example corresponds to the particular negative sentences. So, it can be said that the structure of this type of sentence is ‘not every S is P’ (or ‘some of the S are not P’).

As far as the problem of the different possible translations is concerned, it can be relevant to review, for example, Abelard’s analyses on the correct expression of the particular negative sentences, Sherwood’s equipollences, or the fact that Aristotle does not always resort to the same forms to refer to these kinds of sentences (see, e.g., Parsons, 2008: 163-170). However, what is interesting for this paper is that there is a simpler way to name these kinds of sentences. As it is also well known, medieval logicians used vowels to denote them. Based on the meanings and the two first vowels of the Latin words *affirmo* (I state) and *nego* (I deny), they established these equivalences:

-‘Every S is P’ (or, if preferred, ‘all of the S are P’): sentences of type ‘A.’

-‘Some S is P’ (or, if preferred, ‘some of the S are P’): sentences of type ‘I.’

-‘No S is P’ (or, if preferred, ‘none of the S is P’): sentences of type ‘E.’

-‘Not every S is P’ (or, if preferred, ‘some of the S are not P’): sentence of type ‘O.’

With respect to the relations between these kinds of sentences, it is absolute true that some descriptions of them are to be found in books authored by Aristotle. For example, in *De Interpretatione* 7, 17b16-18, he speaks about contradictory relations (the adverb used by Aristotle is ἀντιφατικῶς), and in *De Interpretatione* 7, 17b20-21, about contrary relations (the adverb used by him is ἐναντίως). Nevertheless, for the aims of this paper, I will only focus on the version of the Square of Opposition generally accepted in the Middle Age, and on the descriptions of the relations given by Peter of Spain in his *Tractatus* (or *Summulae*

logicales), because in this latter book they are exposed in a clear and systematic way.

As it is well known as well, the relations are as follows:

-*Contrariarum lex* [law of contraries]: this law corresponds to the relation between A and E, and establishes that: ... *si una est vera reliqua erit falsa, & non econuerso. Possunt enim ambe esse falsae in contingenti materia, ut omnis homo est albus: & nullus homo est albus* (Peter of Spain, *Tractatus* I, 12F), that is, in English, ‘if one of them is true, the remaining one will be false, and not vice versa. But if the matter is contingent, both of them can be false, as every man is white and no man is white.’

-*Subcontrariarum lex* [law of subcontraries]: this law refers to the relation between I and O, and stipulates that: ... *si una est falsa reliqua erit vera & non econuerso. Possunt enim ambae simul esse vere in contingenti materia: & hoc, quando accidens est separabile: ut quidam homo est albus, quidam homo non est albus* (Peter of Spain, *Tractatus* I, 13B), that is, in English, ‘if one of them is false, the remaining one will be true, and not vice versa. But, if the matter is contingent, both of them can be true at the same time. And this is so when the accident is separable, as some man is white and some man is not white.’

-*Contradictoriarum lex* [law of contradictories]: this law regulates the relations between A and O, and E and I, and states that: ... *si una est vera, reliqua erit falsa, & econuerso: in nulla enim materia possunt ambae esse simul verae vel false, ut omnis homo est animal, quidam homo no est animal,...* (Peter of Spain, *Tractatus* I, 13B), that is, in English, ‘if one of them is true, the remaining one will be false, and vice versa. Both of them can be true or false at the same time in no matter, as every man is an animal and some man is not an animal.’

-*Subalternarum lex* [law of subalternations]: this law corresponds to the relations between A and I, and E and O, and provides that: ... *si universalis est vera sua particularis erit vera & non econuerso. potest enim universalis esse falsa sua particulari existete vera, &, si particularis est falsa, sua universalis erit falsa, & non econuerso* (Peter of Spain, *Tractatus* I, 13F), that is, in English, ‘if the universal one is true, its particular will be true, and not vice versa. But the universal one can be false even if its particular is true, and, if the particular one is false, its universal will be false, and not vice versa.’

As said, these rules are not always easily understood by students, and what is interesting about the mental models theory in this regard is that it predicts the exact relations that can be difficult and indicates the

ways in which such relations can be taught to students to better learn them. Of course, another relevant point about the laws of opposition is the one that arises when empty sets are considered. I will not deal this issue here, but I think that my accounts on the Square of Opposition are enough to present the general methodological strategies that, following the mental models theory, can be used in order to teach Aristotelian logic. By this I mean that, as it can be noted below, the didactic strategies to explain to students the problems related to the cases with empty sets can be drawn easily, even trivially, from the arguments that I will expose.

That said, before starting to analyze the relations included in the square, I will describe in details the theses of the mental models theory that should be taken into account to identify the hard relations and the way to solve their problems.

3. THE MENTAL MODELS THEORY AND THE QUANTIFIED SENTENCES

The mental models theory is a psychological approach with a very wide scope. Therefore, it is not necessary to consider all of its theses here, but only those that refer to the difficulties that will be reviewed. Thus, overall, it can be said that the mental models theory has an important characteristic that distinguishes it from other reasoning theories. It claims that human reasoning is mainly semantic and that human beings, when they reason, resort to semantic models representing the possible scenarios. Thus, each mental model is a representation of the reality consistent with what is stated in the sentences spoken or written. The problem is that working memory and human intellectual abilities in general are limited, and this fact leads to all the possibilities not being taken into account by them. Some of the models are easy to detect, but other models require further intellectual effort.

As far as the quantified sentences are concerned, an interesting paper can be that of Khemlani et al. (2015). In that paper, the models easy to identify are called ‘canonical models.’ On the other hand, the more difficult models are named ‘noncanonical models.’ In this way, it can be said that an important idea of the mental models theory is obvious: most of the errors or mistakes in reasoning are caused by the fact that individuals only pay attention to the canonical models corresponding to sentences or propositions, and not to all of their noncanonical models. So, a first hypothesis, which is in fact my hypothesis in this paper, can be that the relations of the Square of Opposition of Aristotelian logic that are

difficult to teach are those that require to consider noncanonical models to truly understand what they provide.

This may be seen more clearly if the examples given by Khemlani et al. (2015) are reviewed. According to them, in a universe of three elements, the canonical models corresponding to a sentence of type A show that these three elements are both S and P:

S	P
S	P
S	P

This set of models represents three possible cases and, in all of them, the elements have both the property S (the property indicated in the subject) and the property P (the property indicated in the predicate). So, it can be stated that, in that universe, it is obvious that every S is P.

But, if an individual spends more intellectual effort, he may realize that other combinations are possible too and think about alternative noncanonical models. An example of a noncanonical models set corresponding to A offered by Khemlani et al. (2015) is this one:

S	P
not-S	P
not-S	not-P

Note here that the only remaining combination is S and not-P, that is, the one in which a sentence of type A would be false. In this way, the first element has both the property S and the property P. In the second model, S is not had but P does be had. Finally, in the last model, neither S nor P is had. Nevertheless, as it can be noted, with this noncanonical models set, it continues to be correct that every S is P.

As regards I, the canonical models indicated by Khemlani et al. (2015) are the following:

S	P
S	P
S	not-P

Given that the third model informs that S is true and P is false, it cannot be assumed that every S is P. Nonetheless, the two first models, in which both S and P are true, are enough to state that some S is P. However, other combinations are possible in this case too. Khemlani et al.'s (2015) example of noncanonical models is here this one:

S	P
S	not-P
not-S	P

Obviously, the first model allows continuing to state that some S is P.

On the other hand, the canonical models of E can be:

S	not-P
S	not-P
not-S	P

Because there is not a case with both S and P being true, it is absolutely correct that no S is P. Nevertheless, the example of noncanonical models set presented by Khemlani et al. (2015) in this case is as follows:

S	not-P
S	not-P
not-S	P
not-S	not-P

Apart from the fact that this latter set has one more element (the possible scenarios are four), which, of course, involves further cognitive effort, what is important is that this set is also compatible with the sentence 'no S is P,' because there is no scenario in which both S and P happen.

Finally, the canonical models assigned to O by Khemlani et al. (2015) are these ones:

S	not-P
S	not-P
not-S	P

Note that these canonical models are exactly the same as those of E. But, in any case, what is relevant is that this combination also enables to say that not every S is P, since there are two cases, the first one and the second one, in which S is true and P is not. Of course, Khemlani et al. (2015) provide an example of noncanonical models for O, which has four elements as well:

S	not-P
S	not-P
not-S	P
not-S	P

As it can be checked, the two first models are the models that continue to support the idea that not every S is P.

As mentioned, my hypothesis is that the relations of opposition requiring taking noncanonical models into account are the relations more difficult to teach. The reason is that students, as all of the human beings, tend at first to consider only canonical models. Therefore, in my view, teachers' task (or even professors' task at university) is, firstly, to anticipate the relations that can be difficult to understand, and, secondly, to think about good strategies to lead their students to pay attention to the noncanonical models that are needed in such relations.

In this way, in the next pages, I will explain in details which the relations of opposition that, according to the mental models theory, can be hard are. Likewise, I will propose examples that can be used by teachers or professors to achieve that their students identify the combinations of possibilities necessary to understand the problematic relations. To do all of this, I will analyze the different relations separately.

4. MENTAL MODELS AND THE SQUARE OF OPPOSITION

-CONTRARIARUM LEX:

As mentioned, the law of contraries includes three requirements:

- If A is true, then E is false.
- If E is true, then A is false.
- A and E can be false at the same time (when the matter is contingent, i.e., when there is no strict relation between its subject and its predicate).

The first requirement is not difficult. As also indicated, the canonical models of A are:

S	P
S	P
S	P

As it can be noted, none of the three models describes a situation in which S is true and P is false. So, if A is true, E cannot be also true.

Something similar happens with the second requirement. The canonical models of E are the following:

S	not-P
S	not-P
not-S	P

Thus, if this set of combinations is considered, it cannot be said that A is true, since, in fact, none of the scenarios presents a situation in which both S and P are true.

But the case of the third requirement is different. Firstly, to check it, it is necessary to modify the canonical models in order to show that the sentences are false. In principle, this is not hard to do, because just one model with S and not-P transforms a universal affirmative sentence into false, and just one model with S and P transforms a universal negative sentence into false.

Therefore, students only have to think about this combination to make false a sentence of type A:

S	not-P
S	P
S	P

In this combination, only the first model has been handled (P has been denied). However, as said, this circumstance is enough to state that it is not truth that every S is P.

On the other hand, the procedure that must be made to make a sentence of kind E false is very akin. It is enough to modify the first model:

S	P
S	not-P
not-S	P

Indeed, now it is not possible to say that no S is P, because the first model indicates that that is false.

But the problem is that just these two changes do not enable to note that the third requirement is correct, and further intellectual effort is needed. The two latter sets of canonical models must be combined, and that task may not be easy. So, according to the mental models theory, this can be a relation hard to understand for a person that studies Aristotelian logic for the very first time. Individuals are likely to find difficulties to accept that A and E can be false at the same time, and, maybe, that can be made easy only if a good example (of course, related to a contingent matter) is proposed. In this way, an appropriate resource can be to use examples very related to students' general knowledge. Thus, the teacher, or the professor, can ask his (or her) students for, e.g., thinking about the relations between France and Europe. Given that France is in Europe, it cannot be said that 'no European is French.' But, given that France is not the only country in Europe, 'every European is French' is also an incorrect sentence. Probably, examples such as this one will cause students to understand the third requirement without difficulties, and, if the mental models theory is assumed, the reason is obvious. Because of the example indicated and their general knowledge, students do not need to spend a lot of effort to build a noncanonical models set such as, e.g., this one:

European	French
European	not-French
not-European	not-French

The only case that is not valid is that in which an individual is French and is not European. Thus, given that we can expect that our students know these geographical relations, this can be an interesting way to show that two contrary sentences can be false at the same time if the matter is contingent.

-SUBCONTRARIARUM LEX:

In the case of the law of subcontraries, the requirements are:

-If I is false, then O is true.

-If O is false, then I is true.

-I and O can be true at the same time (when the matter is contingent).

As far as the first requirement is concerned, to think about the case in which I is false implies to remove all of the models in which both S and P are true, i.e., if the canonical models of I are taken into account, the two first scenarios. But to do that does not seem to be difficult. It is enough to deny P in those scenarios. In this way, the resulting set would be as follows:

S	not-P
S	not-P
S	not-P

And it is evident that, given this set of models, O is absolutely true, since it is clear that not every S is P. To state this, it is only necessary that there is at least one case of S and not-P, and all of the cases of the set are cases of that kind.

The second requirement is also easy. If O is false, there cannot be cases of S and not-P. Therefore, to assume that O is false, it is only necessary to remove the negation of P in the two first models. Thus, the set that would be considered would be as follows:

S	P
S	P
not-S	P

Obviously, now, none of the cases shows a scenario in which S is true and P is false. So, neither should this requirement be difficult to understand.

In connection with the third requirement, it can be said that, according to the mental models theory, it should be even easier to understand than the other two. The reason is that the canonical models set of I already

allows seeing that cases of S and P and of S and not-P are possible at the same time. In particular, that set includes two cases of S and P and one of S and not-P. It is clear, in this way, that the mental models theory predicts that most of the students will understand the law of subcontraries without difficulties.

-CONTRADICTIONARIUM LEX:

Something similar can be said about the law of contradictories. None of its requirements are problematic, because all of them can be checked quickly by paying attention to just the canonical models. Such requirements are the following:

- If A is true, then O is false.
- If A is false, then O is true.
- If O is true, then A is false.
- If O is false, then A is true.
- If E is true, then I is false.
- If E is false, then I is true.
- If I is true, then E is false.
- If I is false, then E is true.

The first one is absolutely clear because no case of S and not-P can be found in the canonical models set of A. Likewise, if we consider A to be false and change its first canonical model to S and not-P, that change causes O to be true, since it introduces a scenario in which at least one of the S is not P. The third one is also obvious because the canonical models set of O includes two cases of S and not-P (and just one case would be enough to show that A is false). Given that in the fourth one it is necessary to assume that O is false, as indicated in the previous section, its set must be transformed into the following:

S	P
S	P
not-S	P

And, as it can be noted, in this set there are not cases of S and not-P. Consequently, A is true. The fifth requirement is also easy: because in the canonical models set of E there is no scenario in which both S and P are true, if E is true, I cannot be true. On the other hand, if we consider E to be false, we need to change its first model to S and P, which gives us a case in which both S and P are true, makes it true, and enables to check that the sixth requirement is correct as well. As far as the seventh one is concerned, just the first model of the canonical models set of I informs to us that there is at least a case of S and P, and that, therefore, E cannot be true. Finally, if I is false, as explained in the previous section, its set is transformed into this one:

S	not-P
S	not-P
S	not-P

And, according to this set, E is true, because P is false in all of the cases in which S is true.

So, following the mental models theory, the law of contradictories is also a law that should not cause difficulties to the students. Nevertheless, the law of subalternations is different, since it refers to a problematic scenario.

-SUBALTERNARUM LEX:

The requirements of the law of subalternations are these ones:

- If A is true, then I is true.
- If E is true, then O is true.
- If I is true, then A is not necessarily true.
- If O is true, then E is not necessarily true.
- If I is false, then A is false.
- If O is false, then E is false.
- If A is false, then I is not necessarily true.
- If E is false, then O is not necessarily false.

The only problematic requirement here is the fourth one (if O is true, then E is not necessarily true). For this reason, it seems to be opportune to start with the seven remaining and to analyze the fourth one at the end.

The first one is evident because all of the canonical models of A describe situations in which both S and P are true, and, given that I is true with just one of such situations, it is clear that, when A is true, I cannot be false. The second one is even more obvious, since the canonical models set of E is the same as that of O. Therefore, if E is true, O must be true as well. In connection with the third one, it can be said that the fact that the set of I includes a case of S and not-P quickly reveals that A is not necessarily true because I is true.

On the other hand, as said, when I is false, its three scenarios show cases of S being true and P being false, which leads to accept easily that A cannot be true if I is false, i.e., to accept easily the fifth requirement. Nonetheless, I have also indicated the set of O when it is false. In that set, there are two situations in which both S and P are true, which means that, when O is false, E is so too (sixth requirement), since there are cases in which both S and P happen. Likewise, I have explained the way that A can be thought to be false: it is enough to change the first model to another in which S is true and P is not. However, given that the other two canonical models are not modified (and continue to show scenarios in which both S and P occur), it is obvious that the fact that A is false does not imply that I is false, which allows noting that the seventh requirement is also correct without difficulties. Finally, as also indicated, to make E false, we only need to remove the denial of P in its first canonical model. Nevertheless, although this is done, its second canonical model continues to provide a scenario with S being true and P being false. Therefore, E may be false and, at the same time, O may be true.

Thus, the only remaining requirement is the fourth one. As mentioned, the mental models theory predicts that this is hard to understand and to learn, and the reason is clear. As also said, the canonical models sets of O and E are the same, and this circumstance should cause students to have difficulties to note that E can be false even if O is true. Again, it can be very useful to resort to students' general knowledge in order to cause them to build alternative noncanonical models. Continuing with the previous example of France and Europe, one might ask students for thinking about these two sentences:

‘Not every European is French’

‘No European is French’

The first one is a kind O sentence and is true. The second one is a kind E sentence and is false. To note that will not be difficult for students because, as shown, they usually know the geographic relations between

Europe and France. In particular, as also exposed, they know that the possible combinations in this regard are these indicated above and, therefore, that, while it is possible to be European and not to be French at the same time, there are people that are both European and French.

5. CONCLUSIONS

The mental models theory hence can help us anticipate the relations of the Square of Opposition that are likely to cause problems to students. In particular, it predicts that difficulties will appear in the cases of the law of contraries (it will be hard to understand that A and E may be false at the same time) and of the law of subalternations (it will be hard to understand that the fact that O is true does not imply that E is necessarily true as well).

As explained, the problem is that, in these two latter laws, the canonical models are not enough, and students need to consider alternative combinations, i.e., noncanonical models. As also indicated, I think that a good strategy to cause students to take other alternative models into account can be to resort to their general knowledge. Using examples on facts or situations well known by them can make explicit certain combinations of possibilities that need to be thought to understand the Aristotelian relations of opposition, and that are not included in the canonical models sets. Maybe teachers and professors can know that in a more or less intuitive way. However, the framework of the mental models theory is important because it can show which the exact points causing problems or difficulties are, and which the exact models that must be considered are.

In any case, there is a curious fact that deserves to be commented on. The mental models theory is a reasoning approach claiming that human inferential activity is not logical (in the sense that it does not follow syntactic schemata and only analyzes semantic possibilities). Nevertheless, as shown above, the theory can be successfully used to look for (and find) methodological strategies helping to teach logic in an easier and simpler way. Thus, regardless of the discussion on whether or not the mental models theory is right and describes correctly human reasoning, it appears to be evident that this theory can support us in the search of ways of building models representing what the logical relations truly intend to express. And this applies not only to Aristotelian logic, but also maybe to all of the remaining logics, including Stoic logic and the non-classical modern logics.

Furthermore, the theses argued in this paper can also be useful not only for teaching or education, but also for the mental models theory

itself. Indeed, I have developed certain theses of the theory and, as a result, indicated some predictions of it. Perhaps, empirical experiments trying to prove that my theses are correct could provide interesting evidence in favor of the mental models theory. However, in my view, what is most interesting is that, if such experiments achieve positive results, teaching in general, and not only teaching philosophy, could find relevant resources from that theory, since to explain how human beings reason is, in certain sense, to explain how they learn too.

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