A BASIC ANALOGUE OF H-FUNCTION OF TWO VARIABLES

1. INTRODUCTION

The G- and H-functions have been studied extensively by several authors [2,3,7]. Manot and Kalla [4] have extended the H-function in the domain of two variables, whereas Saxena [5] and Srivastava [7] have treated the case of several variables.

The great success of the theory of hypergeometric functions in one and various variables has stimulated the development of a corresponding basic analogue of these functions. Let \( q \) be a parameter which in general shall be restricted to the domain \(|q| < 1\), and

\[
\begin{align*}
(a_q)_q, n &= (1-q)(1-q^2)...(1-q^n), \quad n=1,2,... \\
(a_q)_q, 0 &= 1.
\end{align*}
\]  

(1.2)

Then

\[
\begin{align*}
&\int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} \prod_{i=1}^{r} p_i(z) dz \\
&= \sum_{n=0}^{\infty} \frac{(a_1)_n, (a_2)_n, ..., (a_r)_n}{(q)_n, (b_1)_n, ..., (b_s)_n} z^n \\
&= \sum_{n=0}^{\infty} \frac{(a_1)_n, (a_2)_n, ..., (a_r)_n}{(q)_n, (b_1)_n, ..., (b_s)_n} \frac{z^n}{n!} \\
&= \sum_{n=0}^{\infty} \frac{(a_1)_n, (a_2)_n, ..., (a_r)_n}{(q)_n, (b_1)_n, ..., (b_s)_n} \frac{z^n}{n!} \\
&= \sum_{n=0}^{\infty} \frac{(a_1)_n, (a_2)_n, ..., (a_r)_n}{(q)_n, (b_1)_n, ..., (b_s)_n} \frac{z^n}{n!} \\
&= \sum_{n=0}^{\infty} \frac{(a_1)_n, (a_2)_n, ..., (a_r)_n}{(q)_n, (b_1)_n, ..., (b_s)_n} \frac{z^n}{n!}
\end{align*}
\]  

(1.3)

is a function of \( z \) and of \( r+s+1 \) parameters \( a_1, ..., a_r; b_1, ..., b_s; q \), which reduces to a generalised hypergeometric series \( \sum_{n=0}^{\infty} \frac{(a_1)_n, (a_2)_n, ..., (a_r)_n}{(q)_n, (b_1)_n, ..., (b_s)_n} \frac{z^n}{n!} \) if \( r-s+1 \) and \( q+1 \).

In this paper we introduce a basic analogue of H-function of two variables in the theory of generalised hypergeometric series, which is an extension of the basic H-function defined earlier by Saxena, Modı and Kalla [6]. Some of its main properties are established. The results are then extended to the case of several variables.

2. DEFINITION OF A BASIC H-FUNCTION OF TWO VARIABLES

A basic analogue of H-function of two variables \( [A] \) is defined in terms of a double Mellin-Barnes type integral as :
The integrals converge if

\[ \Re [\log (2\pi) - \log \sin \theta] < 0 \quad \text{and} \quad \Re [\log z_1 - \log z_2] < 0 \]

for large values of \(|t|\) and \(|s|\) on the contours \(\sigma = \sigma_1, \sigma_2\). The result is

\[ \begin{align*}
\Gamma_0 (n_1, n_2, N_1, N_2) & = \sum_{j=M+1}^{N_1} \Gamma_0 (n_1, n_2, j, j) \\
\Gamma_0, \Gamma_1 (P_1, Q_1, P_2, Q_2) & = \sum_{j=M+1}^{N_1} \Gamma_0 (n_1, n_2, j, j)
\end{align*} \]

If we make suitable changes in the parameters in (2.1), it can then give rise to the definitions of the basic analogue of several generalized special functions, such as a \(G\)-function of two variables, Kamp de Feriet's function of two variables, Appell's function of two variables \(F_\alpha, F_\beta, F_\gamma\) and \(F\) and \(\text{Whittaker functions of two variables, etc. For the sake of brevity they are not presented here.}

From the definition (2.1), it is readily seen that

\[ \begin{align*}
\gamma_j & = (n_1, n_2, \alpha_j, \beta_j, \gamma_j) \\
\delta_j & = (n_1, n_2, \alpha_j, \beta_j, \gamma_j)
\end{align*} \]
\[ \sigma_1 + \sigma_2 \text{, } (M_1+1:N_1), (M_2+1:N_2) \]

\[ - (-1) \text{ H } C_{D}, (P_1+1:Q_1), (P_2+1:Q_2) \]

\[ \left( c_{j_i}^j \gamma_j^j, \gamma_j^j \right) \]

\[ (a_{j_1}^j \alpha_{j_2}^r \gamma_j^j, \gamma_j^j) \]

\[ (d_j \delta_{j_1}^j \delta_{j_2}^j, \gamma_j^j) \]

\[ (1,1), (b_j \delta_{j_1}^j \alpha_j, \delta_{j_2}^j) \]

\[ (1,1), (b_j \delta_{j_1}^j \alpha_j, \delta_{j_2}^j) \]

\[ (1,1), (b_j \delta_{j_1}^j \alpha_j, \delta_{j_2}^j) \]

\[ A+1, (M_1+1:N_1), (M_2+1:N_2) \]

3. CERTAIN BASIC INTEGRALS INVOLVING Hq FUNCTION OF TWO VARIABLES:

The following basic integrals will be established.

\[ G(1) \]

\[ G(p) \]

\[ 37 \]

A basic analogue of H-function of several complex variables [5] can be defined analogously. The path of integration $C$ encircles the null-point and also in the usual manner, can be deformed into a loop parallel to the imaginary axis. The proof of these integrals can be developed on considering the integrals as a limit of some definite integrals and by using the residue theorem.

where

\[
\gamma = \gamma (z) = \gamma (z) \gamma (\bar{z})
\]

\[
\alpha = \alpha (z) = \alpha (z) \alpha (\bar{z})
\]

\[
\beta = \beta (z) = \beta (z) \beta (\bar{z})
\]

\[
\delta = \delta (z) = \delta (z) \delta (\bar{z})
\]
and \( G(1 - \alpha \log(zi) - \log \sin \theta) \) for \( j \in \{1, \ldots, N_i\} \) and \( i \in \{1, \ldots, n\} \) lie to left of the contours.

An empty product is interpreted as unity. The poles of the integrand are assumed to be simple.

The integrals converge if \( \Re\{\alpha \log(\alpha) - \log \sin \theta\} < 0 \) for large values of \( |\alpha| \) on the contours i.e. \( \beta(\theta) \leq \log |\alpha| \) for \( 1 = 1, 2, \ldots, n \).

Finally, it is interesting to observe that the results (2.3), (2.1), (3.2) and (3.3) can be extended to a basic analogue of the H-function of several variables defined in this section.

REFERENCES


